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Work Interaction in Quasi-One-Dimensional Flows

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Theoretical work addition is derived in a form analogous to that of classical Rayleigh heat addition. From this derivation it is evident that work terms can be added to the Navier–Stokes momentum and energy equations representing work interaction. A second-law-based effectiveness parameter controlling the ratio of realizable work interaction to the ideal work interaction is required for nonideal analysis. Thus, the theoretical limits on work interaction are parameterized, and a method is developed for easily modeling isentropic and nonisentropic work interaction in computational fluid dynamics. This fundamental theoretical approach is independent of the method employed for the work interaction, that is, turbomachinery, magnetohydrodynamics, etc. . . .

Nomenclature

Cp = specific heat at constant pressure
Cv = specific heat at constant volume
E_t = total energy
e = internal energy
e_c = traditional polytropic compressor efficiency
e_t = traditional polytropic turbine efficiency
$\boldsymbol{f}$ = body forces
$\boldsymbol{f}_{\dot{w}}$ = force due to work
h = static enthalpy
M = Mach number
$\dot{m}$ = mass flow rate
P = pressure
$\boldsymbol{q}$ = heat transfer
$\dot{q}$ = heat addition or removal per mass unit
R = ideal gas constant
S = one-dimensional area
$\boldsymbol{S}$ = surface coordinate
s = entropy
T = temperature
t = time
u = one-dimensional velocity
V = volume
$\boldsymbol{V}$ = velocity vector
$\dot{w}$ = work addition or removal per mass unit
γ = ratio of specific heats, Cp/Cv
$\Delta\xi$ = differential quantity where ξ is generic
$\delta\xi$ = differential volumetric quantity where ξ is generic
η_c = traditional adiabatic compressor efficiency
η_t = traditional adiabatic turbine efficiency
η_w = work-effectiveness parameter
ξ_{is} = isentropic condition where ξ is generic
ξ_t = total condition where ξ is generic
$\bar{\xi}$ = averaged quantity where ξ is generic
π_c = total pressure ratio across the compressor
π_t = total pressure ratio across the turbine
ρ = density
ϱ = density ratio
τ = shear stress
τ_c = total temperature ratio across the compressor
τ_t = total temperature ratio across the turbine
τ_λ = ratio of burner stagnation enthalpy to freestream enthalpy

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Introduction

RAYLEIGH heat addition[1–3] allows the calculation of the effect of heat addition (or removal) with the minimum allowable gain in entropy in a one-dimensional constant-area duct where no other interactions, that is, friction or work, are being considered. This fundamental relationship has proved instrumental in advancing the areas of thermodynamics in general and propulsion in particular. Noting the simplicity and value of the closed-form heat/entropy relationship, a similar closed-form relationship is developed for one-dimensional constant-area work interaction. As in the case of Rayleigh flow, there can be no other interactions in the work region. Unlike heat interaction, work interaction does not have an inherent entropy increase, that is, the ideal work interaction is isentropic. This enables the rapid calculation of the maximum theoretical performance of any work-interaction device. In addition, the effect of entropy on the amount of work realized for a net work interaction is examined.

The closed-form constant-area Rayleigh relationship is frequently employed as a simple combustor model in basic cycle analysis.[4] The corresponding closed-form work equations are more valuable as a conceptual device rather than an application-oriented device because most engine configurations have significant area variation in work zones with a goal toward maintaining a constant axial velocity through turbomachinery stages with the possible exception of magnetohydrodynamic (MHD) propulsion systems.[5–7] For application-oriented devices a work-interaction term is added to the quasi-one-dimensional Euler equations enabling work interaction in conjunction with area change, Fanno flow, and Rayleigh flow. Additionally, a work effectiveness is defined to enable nonisentropic work interaction. By the use of fundamental theory, the maximum performance for a given amount of energy addition in the form of work is parameterized. The importance of the limit is that it holds regardless of the manner in which work is added, that is, turbomachinery, MHD, etc. . . . Moreover, extension of the closed-form work equations to a source term for the quasi-one-dimensional Euler equations enables the modeling of any method of work interaction including, but not restricted to, turbomachinery and MHD using cycle analysis[4] and computational fluid dynamics (CFD). The extension of the source term to multiple dimensions for CFD modeling is straightforward.

In this paper, the motivation behind developing the one-dimensional work relations is explained. Both isentropic and nonisentropic equations are developed, and the trends observed are compared to those observed for Rayleigh flow leading to the idea of energy choking. The nonisentropic considerations lead to the definition of a work-effectiveness parameter and the subsequent extension to a source term for the Euler equations. Next, the relationship between the work-effectiveness parameter and the traditional turbomachinery adiabatic and polytropic efficiencies is examined. As a culmination of the material introduced, the quasi-one-dimensional Euler equations with both Rayleigh and work source terms are applied to a simple flowfield with the traditional turbojet engine components (compressor, burner, and turbine).

Quasi-One-Dimensional Work Addition Motivation and Theory

The motivation behind the development is discussed along with basic theoretical considerations and observations. In investigating work addition and removal, that is, compressors, turbines and MHD accelerators and decelerators, it is beneficial to be able to predict theoretically performance in both subsonic and supersonic regimes. This is routinely done for heat when modeling the combustion process as a constant-area duct with no other interactions and is known as Rayleigh heat addition.

By noting of the usefulness of the Rayleigh heat addition, the obvious work analogy is sought. From the definition of entropy, $\Delta s = \Delta Q / T$, it is obvious that ideal work interaction is isentropic. Both mass and energy must be conserved, but work leads to a change in streamthrust. Thus, the obvious approach is to combine the equations for entropy, conservation of mass, and conservation of energy to determine the net change in streamthrust due to work. The results are extended to nonideal analysis by combining nonisentropic equations with conservation of mass and energy. This leads to the definition of a work effectiveness, which is simply the ratio of the actual work-interaction to the ideal work-interaction with the fluid. The work-effectiveness can be calculated for a specific increase in entropy, but it can also be treated as an input. By the use of the work-effectiveness parameter definition, a source term accounting for real work effects is added to the Euler equations creating a powerful tool for numerical analysis.

Note that the inclusion of work terms in the energy equation is common. An energy equation including work and heat terms is available,[8,9] but no effort is made to couple the energy equation to the continuity and momentum equations in a form analogous to Rayleigh flow.

In addition, note that the approach to quasi-one-dimensional work interaction bears some similarity to actuator disk theory, but has some important distinctions. Both occur, in the limit of discretization, over an infinitesimally thin spacial distance. The general assumptions in actuator disk theory of incompressible flow and circular cross section[10] are not present, even in the one-dimensional closed-form equations. Moreover, the generalization to the Euler equation source term allows concurrent heat, work, and viscous interaction with the flowfield without the constraint of constant area. The use of the Euler equations as a basis allows for discontinuities, that is, shock structures, to be present in regions with work interaction.

The following sections provide the closed-form one-dimensional, that is, constant-area, equations for supersonic and subsonic work interaction for both isentropic and nonisentropic work processes. From the derivations, the usefulness of a work-effectiveness parameter becomes evident. Thus, the work-effectiveness parameter is compared with the traditional adiabatic and polytropic turbomachinery efficiencies. Next, the work Euler source term is obtained. The end result is the demonstrated ability to model work and heat engine components.

Isentropic and Nonisentropic Work Analysis

To examine the fundamental effects of work interaction on flowfields, the integral form Navier–Stokes equations for conservation of mass, momentum, and energy are introduced:

$$\frac{\partial}{\partial t}\int_V \rho \,\mathrm{d}V = -\int_S \rho \boldsymbol{V}\cdot \mathrm{d}\boldsymbol{S} \tag{1}$$

$$\frac{\partial}{\partial t}\int_V \rho \boldsymbol{V}\,\mathrm{d}V + \int_S (\rho\boldsymbol{V}\cdot \mathrm{d}\boldsymbol{S})\boldsymbol{V} = \int_V \rho \boldsymbol{f}\,\mathrm{d}V - \int_S P\,\mathrm{d}\boldsymbol{S} + \int_S \tau\,\mathrm{d}\boldsymbol{S} + \int_V \rho \boldsymbol{f}_{\dot{w}}\,\mathrm{d}V \tag{2}$$

$$\frac{\partial}{\partial t}\int_V \rho\left(e + \frac{1}{2}\boldsymbol{V}^2\right)\mathrm{d}V + \int_S \rho\left(e+\frac{1}{2}\boldsymbol{V}^2\right)\boldsymbol{V}\cdot\mathrm{d}\boldsymbol{S} = \int_S \boldsymbol{q}\cdot\mathrm{d}\boldsymbol{S} + \int_V \rho(\dot{q}+\dot{w})\,\mathrm{d}V + \int_V(\rho\boldsymbol{f}\,\mathrm{d}V)\cdot\boldsymbol{V} - \int_S (P\,\mathrm{d}\boldsymbol{S})\cdot\boldsymbol{V} + \int_S(\tau\,\mathrm{d}\boldsymbol{S})\cdot\boldsymbol{V} \tag{3}$$

In the integral form, the Navier–Stokes equations allow work to be done directly to the flow without modeling boundary interaction, the traditional (physical) manner used to obtain work interaction.

For the fundamental analysis, the Navier–Stokes equations for conservation of mass and energy are reduced to a one-dimensional, steady-state inviscid form, and the assumption of ideal gases is made. Because for one-dimensional (constant-area) isentropic work interaction through any work-interaction device both energy and mass are conserved, the standard isentropic relations [Eq. (6)] are used to close the simplified mass and energy equations:

$$\rho_1 u_1 = \rho_2 u_2 \tag{4}$$

$$CpT_1 + \tfrac{1}{2}u_1^2 + \delta\dot{w} = CpT_2 + \tfrac{1}{2}u_2^2 \tag{5}$$

$$(T_2/T_1)^{1/(\gamma-1)} = \rho_2/\rho_1 = (P_2/P_1)^{1/\gamma} \tag{6}$$

By the definition of ϱ such that

$$\varrho = \rho_1/\rho_2 \tag{7}$$

and the combination of the simplified equations for conservation of mass and momentum and the isentropic relations, the following formulas are provided relating work interaction and density ratio ϱ, assuming ideal-gas relations:

$$\left(\frac{1}{\varrho}\right)^{\gamma-1} = \frac{2CpT_1 + u_1^2 + 2\delta\dot{w}_{\mathrm{is}} - (u_1\varrho)^2}{2CpT_1} \tag{8}$$

Equation (8) is a function of upstream temperature. The equation can be cast such that it is a function of any of the upstream state variables. Because the objective of work interaction in turbomachinery is to alter pressure, the equation using the upstream pressure is provided as

$$\left(\frac{1}{\varrho}\right)^{\gamma-1} = \left[2CpT_1 + u_1^2 + 2\delta\dot{w}_{\mathrm{is}} - (u_1\varrho)^2\right]\frac{(\gamma-1)\rho_1}{2\gamma P_1} \tag{9}$$

A Mach number-based formulation, relating work interaction to outgoing Mach number, is also provided,

$$\left(1 + \frac{\gamma-1}{2}M_1^2 + \frac{\delta\dot{w}_{\mathrm{is}}}{CpT_1}\right) = \left(\frac{M_1}{M_2}\right)^{2(\gamma-1)/(\gamma+1)}\left(1+\frac{\gamma-1}{2}M_2^2\right) \tag{10}$$

Because pressure is a key variable in turbomachinery, an equation based on the static pressure ratio is also presented:

$$\frac{P_2}{P_1} = \left(1 + \frac{u_1^2}{2CpT_1} + \frac{\delta\dot{w}_{\mathrm{is}}}{CpT_1} - \left(\frac{P_2}{P_1}\right)^{-2/\gamma}\frac{u_1^2}{2CpT_1}\right)^{\gamma/(\gamma-1)} \tag{11}$$

Various other forms of the isentropic work equation are obtainable from algebra and the application of the various isentropic relations.

Table 1 Rayleigh heat addition

Flow property	Subsonic	Supersonic
Mach number	Increases	Decreases
Velocity	Increases	Decreases
Pressure	Decreases	Increases
Temperature	Increases for $M_1 < \gamma^{-1/2}$, decreases otherwise	Increases
Total pressure	Decreases	Decreases
Total temperature	Increases	Increases

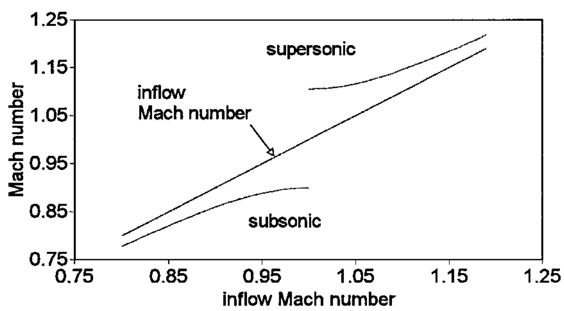

Fig. 1 Effect of work addition on Mach number in subsonic and supersonic flows (based on $\delta\dot{w}$ = 1000 J/kg).

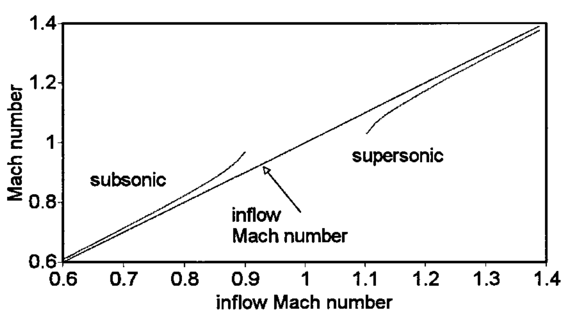

Fig. 2 Effect of work extraction on Mach number in subsonic and supersonic flows (based on $\delta\dot{w}$ = − 900 J/kg).

It is instructive to examine the effect of work interaction on Mach number, that is, what effect does ideal work addition have on exit Mach number? The effect of work addition on both subsonic and supersonic flows is shown in Fig. 1. From Fig. 1, it is evident that work addition decreases the subsonic and increases the supersonic Mach number of the flow. This is exactly opposite the trend seen in Rayleigh heat addition (see Table 1). Figure 1 is generated by varying the inflow Mach number and solving the isentropic work relationships[Eq. (10)] with a δw = 1000 J/kg for the outflow Mach number.

The effect of work extraction on subsonic and supersonic flows is shown in Fig. 2. From Fig. 2, it is evident that work removal drives the flow Mach number toward unity. The work-removal effect on Mach number is opposite the heat-removal result obtained from Rayleigh flow analysis (see Table 1). Figure 2 is generated by varying the inflow Mach number and solving Eq. (10) with a $\delta w = -900$ J/kg for the outflow Mach number.

From these observations, the term "energy choking" is introduced, which is inclusive of both work and heat, rather than 'thermal choking,' which is used to describe the upper limit (due to choking of the flow) on allowable energy addition by heat. In the case of work, it is energy-extraction that drives the flow to choke. In both cases, continuing the energy transfer process after this sonic point is reached, that is, extracting more work or adding more heat, would necessarily force the upstream conditions to change.

The general isentropic work-interaction trends for Mach number, velocity, pressure, temperature, density, total pressure, and total temperature are summarized in Tables 2 and 3. For completion and comparison, Rayleigh trends[2] are summarized in Table 1. Note that work-removal trends parallel heat-addition trends for Mach number, velocity, and static conditions. The main difference is that in Rayleigh flow total pressure and total temperature move in opposing directions whereas for work-removal they increase or decrease together being linked by the isentropic relations. Unremarkably, work-addition trends are opposite work-removal trends.

Table 2 Work addition

Flow property	Subsonic	Supersonic
Mach number	Decreases	Increases
Velocity	Decreases	Increases
Pressure	Increases	Decreases
Temperature	Increases	Decreases
Density	Increases	Decreases
Total pressure	Increases	Increases
Total temperature	Increases	Increases

Table 3 Work removal

Flow property	Subsonic	Supersonic
Mach number	Increases	Decreases
Velocity	Increases	Decreases
Pressure	Decreases	Increases
Temperature	Decreases	Increases
Density	Decreases	Increases
Total pressure	Decreases	Decreases
Total temperature	Decreases	Decreases

Now that the isentropic work relations have provided important information on general trends, the next step is to examine the effect of losses (entropy gains) on work-interaction. By the utilization of the entropy relations for nonisentropic flow and the conservation of mass and energy, the one-dimensional work-interaction formulas with losses are developed. The derivations involve substituting conservation of mass and nonisentropic relations for ratios in the energy equation. It is beneficial to transform the ideal-gas entropy relations

$$\Delta s = Cp \ln(T_2/T_1) - R \ln(P_2/P_1) \tag{12}$$

$$\Delta s = Cv \ln(T_2/T_1) + R \ln(\rho_1/\rho_2) \tag{13}$$

using the natural logarithm properties and the ideal-gas equation to the following equations:

$$T_2/T_1 = [\exp(\Delta s/R)(P_2/P_1)]^{(\gamma-1)/\gamma} \tag{14}$$

$$T_2/T_1 = [\exp(\Delta s/R)(\rho_2/\rho_1)]^{\gamma-1} \tag{15}$$

$$P_1/P_2 = \exp(\Delta s/R)(T_2/T_1)^{-\gamma/(\gamma-1)} \tag{16}$$

$$\rho_1/\rho_2 = \exp(\Delta s/R)(T_2/T_1)^{-1/(\gamma-1)} \tag{17}$$

Furthermore, note that the nonisentropic relations are related to Mach number via the conservation of mass. The relationship can be written as

$$T_2/T_1 = [M_2/M_1 \exp(\Delta s/R)]^{-2(\gamma-1)/(\gamma+1)} \tag{18}$$

Combining the preceding entropy relations with the conservation of mass and energy in an analogous fashion to the isentropic coupling yields closed-form work-interaction equations that, while still one-dimensional, can account for losses (entropy gains) incurred during the work process. As in the isentropic case, given the work interaction and entropy increase, the new density ratio can be obtained:

$$\exp\left(\frac{\Delta s}{R}\right) = \left[\frac{2CpT_1 + u_1^2 + 2\delta\dot{w} - (u_1\varrho)^2}{2CpT_1}\right]^{1/(\gamma-1)} \varrho \tag{19}$$

Again, Eq. (19) can be cast in terms of the upstream pressure rather than the upstream temperature,

$$\exp\left(\frac{\Delta s}{R}\right) = \left[\left(2CpT_1 + u_1^2 + 2\delta\dot{w} - (u_1\varrho)^2\right)\frac{(\gamma-1)\rho_1}{2\gamma P_1}\right]^{1/(\gamma-1)}\varrho \tag{20}$$

Casting in terms of entrance and exit Mach numbers, yields

$$\left(1 + \frac{\gamma-1}{2}M_1^2 + \frac{\delta\dot{w}}{CpT_1}\right) = \left(\frac{M_2}{M_1\exp(\Delta s/R)}\right)^{-2(\gamma-1)/(\gamma+1)}\left(1 + \frac{\gamma-1}{2}M_2^2\right) \tag{21}$$

Finally, because pressure is a key variable in turbomachinery, an equation based on the static pressure ratio is also presented:

$$\left(1 + \frac{u_1^2}{2CpT_1} + \frac{\delta\dot{w}}{CpT_1}\right) = \exp\left[\frac{\Delta s(\gamma-1)}{\gamma R}\right]\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} + \left(\frac{P_2}{P_1}\right)^{-2/\gamma}\frac{\exp[2\Delta s(\gamma-1)/\gamma R]u_1^2}{2CpT_1} \tag{22}$$

As was the case for the isentropic relations, the nonisentropic equations can be recast in any variable desired by performing algebraic manipulation and applying the appropriate entropy relations.

Efficiency Evaluation

Obviously, there is a difference in the amount of work realized between isentropic and nonisentropic work processes. Traditional efficiencies of turbines and compressors[3] defined as

$$\eta_c = \frac{\text{ideal work interaction for a given total pressure ratio}}{\text{actual work interaction for a given total pressure ratio}} \tag{23}$$

$$\eta_t = \frac{\text{actual work interaction for a given total pressure ratio}}{\text{ideal work interaction for a given total pressure ratio}} \tag{24}$$

are based on a constant total pressure ratio, which is not very convenient because the work relations are based on a specified work input and entropy gain.

To that end, η_w is introduced as a work-effectiveness parameter. More precisely, η_w is a second-law-based efficiency relating the actual work-interaction obtained to the ideal work-interaction obtainable:

$$\eta_w = \frac{\text{actual work interaction with the fluid}}{\text{ideal work interaction with the fluid}} \tag{25}$$

Expressing relation (25) in equation form yields the following expression for the steady-state condition, where the numerator represents the actual change in force realized and the denominator represents the ideal work interaction:

$$\eta_w = \frac{\left(P_2 + \rho_2 u_2^2\right)S_2 - \left(P_1 + \rho_1 u_1^2\right)S_1 - \bar{P}\Delta S}{\bar{\rho}\bar{S}\delta\dot{w}} \tag{26}$$

For clarity the work-effectiveness parameter, η_w, is conceptually compared to the traditional efficiency measures. Traditional measures of efficiency are always less than one. For a real compressor, more work than the ideal is necessary to achieve a total pressure rise because losses must be overcome. On the other hand, for a real turbine less work is produced for a given total pressure drop than for an ideal turbine.

Conversely, the work-effectiveness parameter, η_w, for a compressor is less than one, whereas for a turbine η_w is greater than one. The losses associated with the turbine extracting work from the flow result in the total pressure drop being greater than the ideal (isentropic) total pressure drop associated with the work-removal; therefore, the work-effectiveness for nonisentropic work-removal is greater than one. For work addition, losses serve to reduce the actual work-interaction with the flowfield; thus, the traditional trend of a decreasing work-effectiveness parameter is associated with increasing losses. The work-effectiveness parameter, η_w, provides a simple, straightforward measure of the relationship between ideal and nonideal work-interaction.

Because the traditional efficiencies, η_c and η_t, and the work-effectiveness term, η_w, can be calculated for any turbomachine, it is instructive to examine the relationship between the traditional efficiencies and the work-effectiveness parameter for both the cases of work addition and extraction. For a fluid compression device modeled with work-addition of 100 kJ/kg and inflow conditions of 330 m/s (M_∞ =0.97), 101,325 Pa, and 1.23 kg/m^3 and entropy varying from 0 to 200 J/kg · K, the decreasing work-effectiveness due to increasing entropy is shown in Fig. 3. The variation of work-effectiveness with entropy is approximately linear. The corresponding relationship between work-effectiveness and exit Mach number is shown in Fig. 4. Poor work-effectiveness serves to drive the exit Mach number rapidly toward one in a work-addition device. This occurs because inefficiencies in work-addition translate into heat-addition. Thus, poor work-addition trends move toward Rayleigh trends as the work effectiveness decreases. Totally ineffective work-addition indicates all work is lost to heat and is described by Rayleigh flow. The variation of compressor total pressure ratios π_c and total temperature ratios τ_c with work effectiveness is provided in Fig. 5. The total temperature remains constant for a given amount of work (energy) interaction, but the total pressure rise is a result of a realized gain in momentum (streamthrust) governed by the work-effectiveness term, η_w.

Note that the work-effectiveness term, η_w, is not a typical measure of compressor efficiency; the corresponding adiabatic, η_c, and polytropic, e_c, compressor efficiencies,[3]

$$\eta_c = \frac{\pi_c^{(\gamma-1)/\gamma} - 1}{\tau_c - 1} \tag{27}$$

$$e_c = \frac{[(\gamma-1)/\gamma]\ln(\pi_c)}{\ln\left\{\left[\pi_c^{(\gamma-1)/\gamma} - 1\right]/\eta_c + 1\right\}} \tag{28}$$

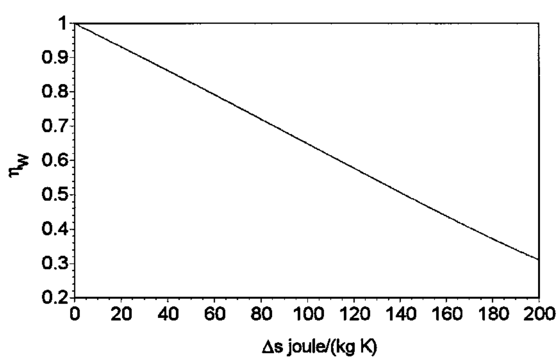

Fig. 3 Effect of increasing entropy on compressor work effectiveness η_w.

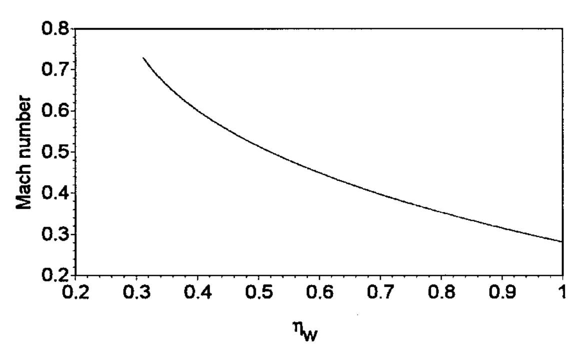

Fig. 4 Relationship between compressor work effectiveness η_w and exit Mach number for the same energy input.

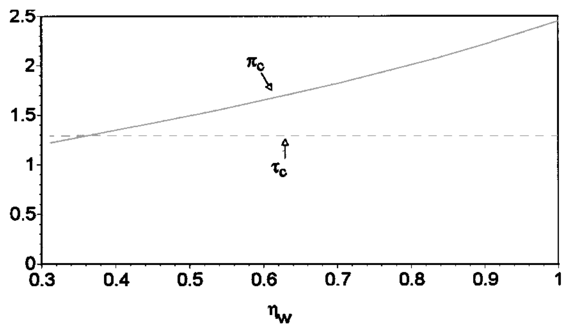

Fig. 5 Relationship between compressor work effectiveness η_w and the resulting total pressure π_c and total temperature τ_c ratios through the compressor.

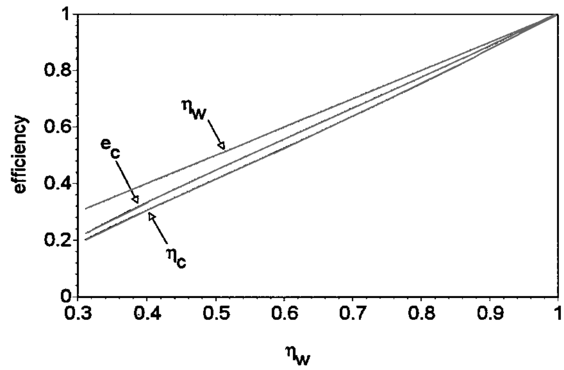

Fig. 6 Relationship between compressor work effectiveness η_w and the adiabatic compressor efficiency η_c and the polytropic compressor efficiency e_c.

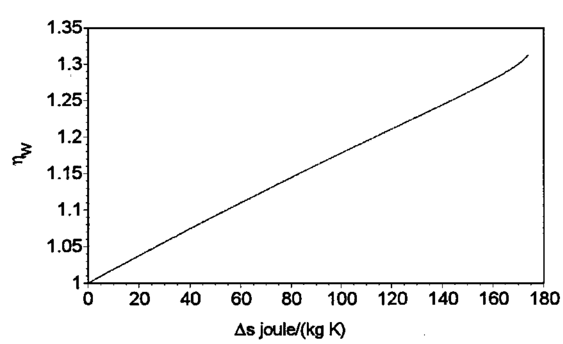

Fig. 7 Effect of increasing entropy on turbine work effectiveness η_w.

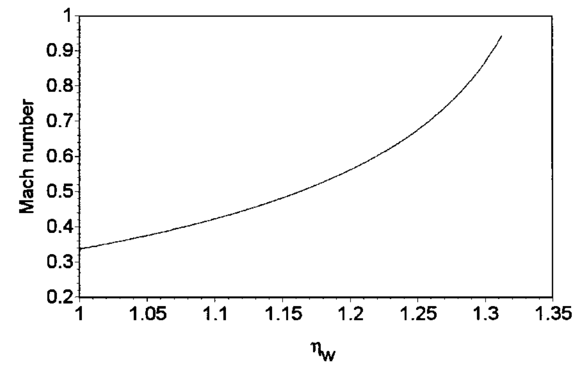

Fig. 8 Relationship between turbine work effectiveness η_w and exit Mach number for the same energy input.

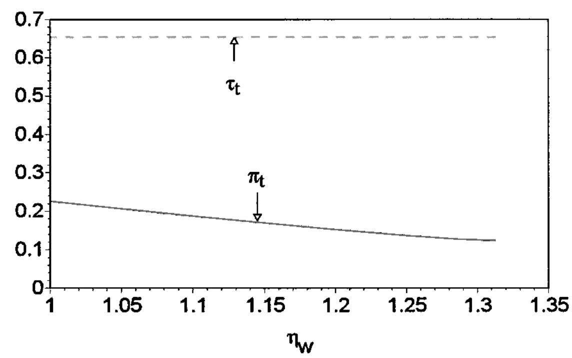

Fig. 9 Relationship between turbine work effectiveness η_w and the resulting total pressure π_t and total temperature τ_t ratios through the turbine.

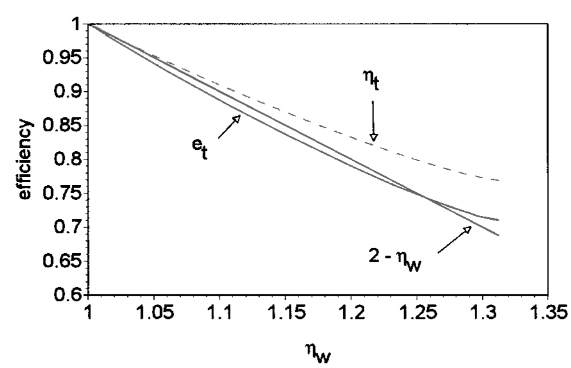

Fig. 10 Relationship between turbine work effectiveness η_w and the adiabatic turbine efficiency η_t and the polytropic turbine efficiency e_t.

are provided in Fig. 6. Note that the adiabatic and polytropic efficiencies and the work-effectivenessare all equal to one for the ideal case. The traditional efficiency measures decrease with increasing losses in a nearly parallel fashion and more rapidly than the work-effectiveness parameter.

Now, analyzing a turbine extracting 100 kJ/kg of work with inflow conditions 30 m/s ($M_\infty = 0.088$), 101,325 Pa, and 1.23 kg/m^3 and entropy varying from 0 to 175 J/kg · K, the resulting relationship between entropy and work effectiveness is shown in Fig. 7. The relationship is approximately linear, with some nonlinearities developing for high entropy values.

Exit Mach number variation with turbine work effectiveness is shown in Fig. 8. As the turbine becomes less efficient, more of the work interaction reenters the flow in the form of heat, thus, increasing the exit Mach number. As before the more inefficient the work interaction, the more the results resemble those from Rayleigh analysis.

The effect of turbine work-effectiveness on the total pressure ratio, π_t, achieved and the total temperature ratio, τ_t, across the turbine is shown in Fig. 9. Again, total temperature remains constant while the total pressure ratio across the turbine drops, reflecting more losses as the efficiency drops.

The turbine work-effectiveness is also related to the more commonly used adiabatic, η_t, and polytropic, e_t, turbine efficiencies via[3]

$$\eta_t = \frac{1 - \tau_t}{1 - \pi_t^{(\gamma - 1)/\gamma}} \tag{29}$$

$$e_t = \frac{\ln(\tau_t)}{\ln[1 - (1 - \tau_t)/\eta_t]} \tag{30}$$

The results are shown in Fig. 10. As before for the work-removal case, the polytropic and adiabatic efficiencies match the work-effectiveness parameter for the ideal case and follow the same general trends. Note that for inefficient work the work-effectiveness increases more rapidly than the traditional efficiencies decrease.

Application

With the definition of a work rather than pressure-based effectiveness parameter, all of the tools needed to model realistic configurations are obtained. To this end, the integral form of the Navier–Stokes equations for conservation of mass, momentum, and energy presented earlier as Eqs. (1–3) are reduced to the quasi-one-dimensional Euler equations,[11,12] with a source vector that includes the work term. The equation is presented in matrix form,

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho S\\ \rho u S\\ E_t S\end{bmatrix}+\frac{\partial}{\partial x}\begin{bmatrix}\rho u S\\ (P+\rho u^2)S\\ u(E_t+P)S\end{bmatrix}=\begin{bmatrix}0\\ \bar{P}\dfrac{\Delta S}{\Delta x}+\bar{\rho}\bar{S}\eta_w\dfrac{\delta\dot{w}}{\Delta x}\\ \dot{m}\left(\dfrac{\delta\dot{q}+\delta\dot{w}}{\Delta x}\right)\end{bmatrix} \tag{31}$$

with $\delta\dot{q}$ and $\delta\dot{w}$ in units of joule per kilogram, where

$$E_t = P/(\gamma-1)+\tfrac{1}{2}\rho u^2 = \rho\left(h+\tfrac{1}{2}u^2\right)-P \tag{32}$$

and $\dot{m}$ is a constant.

The Euler equations with the work and heat interaction source terms can be used to model numerous engine applications. As an example, a simple flowfield containing the components of a turbojet, that is compressor, combustor, and turbine, is modeled using an implicit flux-split algorithm with a compressor work effectiveness η_w of 0.9 and a turbine work effectiveness η_w of 1.2. Ideal mechanical efficiency is assumed between turbomachine components.

For the model, total conditions at the engine inlet are 266 K and 39,619 Pa yielding a converged mass flow rate for the subsonic entrance boundary condition of 4.05 kg/s and a Mach number of 0.582. The exit boundary condition is supersonic, therefore, the flow is extrapolated.

The engine contains the traditional components of a turbojet. In the burner, from 0.3 to 0.6 m, 1.0×10^3 kJ/kg of heat is added corresponding to a τ_λ value of 5.58. The compressor is located from 0.1 to 0.3 m and adds 131.75 kJ/kg of work with a work effectiveness $\eta_w=0.9$. The turbine is located from 0.6 to 0.8 m and removes 131.75 kJ/kg of work with a work effectiveness $\eta_w=1.2$.

The results of the CFD analysis yield a compressor total pressure ratio $\pi_c=3.5$ and total temperature ratio $\tau_c=1.5$. For the turbine, postprocessing the solution yields a total pressure ratio $\pi_t=0.65$ and total temperature ratio $\tau_c=0.91$.

Using the aforementioned engine parameters in the CFD analysis, Fig. 11 is generated, which shows the effects of work and heat interaction on the Mach number profile, shown on the left axis, through the engine. The engine area is shown on the right-hand axis to clarify the Mach number trends. The trends concur with the expected results. For further clarity, the pressure and temperature variation through the engine components is provided in Fig. 12. As expected, the total pressure and temperature rise through the compressor and drop through the turbine. In the combustor, the total temperature increases and the total pressure drops as predicted by Rayleigh analysis.

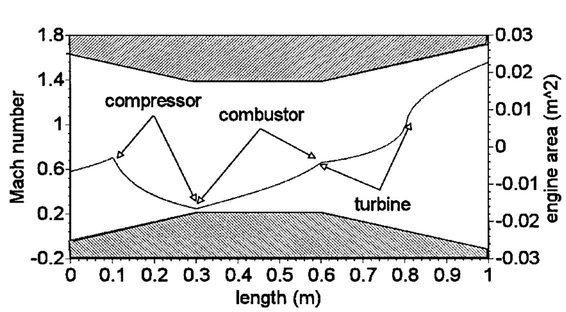

Fig. 11 Mach number profile through the turbojet showing the regions of work and heat interaction; engine geometry (area in square meters) is shown by the bounding lines.

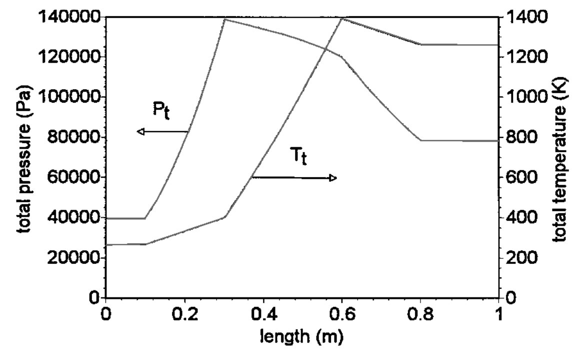

Fig. 12 Total temperature T_t and total pressure P_t profiles through the turbojet.

With the inclusion of the work source term in the Euler equations it has been demonstrated, via a simple model, that CFD can effectively model work interaction directly. There is no restriction to the additional source terms that can be concurrently modeled. Thus, the robustness and usefulness of the derived work relations is demonstrated.

Conclusions

In the same manner as Rayleigh heat addition indicates a thermal limit on an engine, referred to as thermal choking, the isentropic one-dimensional constant-area work relations demonstrate that there is a fundamental limit on work-extraction in supersonic and subsonic flows. This limit holds regardless of the means used to perform the work-interaction. As more work is extracted, the Mach number is driven toward unity. The all-encompassing term "energy choking" is introduced to cover flow driven toward choking (Mach 1) by either heat or work interaction. The simple work relations also indicate work-addition is always safe from an "energy choking" standpoint.

One-dimensional isentropic work equations are presented in closed form. The equations are extended to general nonisentropic closed forms. From these relations, it is possible to construct the terms for the Euler and Navier–Stokes equations to account for the effects of nonisentropic work-interaction.

Coupling of nonisentropic equations to the closed-form work relations increases their applicability, but it is noted that few work-interaction devices (with the possible except of MHD) have any semblance of constant area. The true robustness of the work relation is realized when the Euler source term with the work-effectiveness parameter is introduced. The work-effectiveness parameter, η_w, provides the ratio of actual to ideal work-interaction with the fluid. For a compressor, the actual work-interaction (change in total pressure) is less than the ideal, whereas, conversely, for the turbine the actual work-interaction is greater than the ideal. Flow losses (friction, heat, etc.) serve to reduce the effect of adding work on total pressure rise in the work-addition region (compressor) while causing additional total pressure losses in the work-extraction region (turbine). Coupling work interaction to the Euler equation as a source term enables concurrent modeling of work-interaction along with any additional source terms desired, the most obvious being heat and friction. Although the closed-form equations are instructive from a fundamental understanding viewpoint, the true power of the relationship lies in the relatively straightforward extension to the multidimensional Euler equations that can model flow with concurrent flow interaction devices. As a further extension work-interaction zones can vary in position with time, thus modeling dynamic interaction.

References

[1]Shapiro, A. H., *The Dynamics and Thermodynamics of Compressible Fluid Flow*, 1st ed., Vol. 1, Wiley, New York, 1953, pp. 159–262.

[2]Anderson, J. D., *Modern Compressible Flow, with Historical Perspective*, 1st ed., McGraw–Hill, New York, 1982, pp. 67–78, 361–590.

[3]Oates, G. C., *Aerothermodynamics of Gas Turbine and Rocket Propulsion. Revised and Enlarged*, 1st ed., AIAA, Washington, DC, 1988, pp. 19–62.

[4]Riggins, D. W., "Effects of Combustion and Ireversibilities on the Performance of High Speed Engines (NASA Grant NAG1-1189)," *SCRAMjet Engine Performance Analysis, Evaluation, and Optimization, Proceedings of the JANNAF Propulsion and Joint Subcommittee Meeting Scramjet Performance Workshop*, Dec. 1996, Chap. 2.

[5]Vasil'eva, R. V., Erofeev, A. V., Zuev, A. D., Kuranov, A. L., Lapushkina, T. A., and Mirshanov, D. N., "Experience in MHD Conversion of the Supersonic Air Flow Energy into Electrical Power," *Journal of Technical Physics*, Vol. 39, No. 2, 1994, pp. 143–149.

[6]Bityurin, V. A., Lineberry, J. T., Potebnia, V. G., Alferov, V. I., Kuranov, A. L., and Sheikin, E. G., "Assessment of Hypersonic MHD Concepts," AIAA Paper 97-2393, June 1997.

[7]Bruno, C., Murthy, S. B. N., and Czysz, P. A., "Electro-Magnetic Interactions in Hypersonic Propulsion Systems," AIAA Paper 97-3389, 1997.

[8]Munson, B. R., Young, D. F., and Okiishi, T. H., *Fundamentals of Fluid Mechanics*, 3rd ed., Wiley, New York, 1998, pp. 257–283.

[9]White, F. M., *Fluid Mechanics*, 3rd ed., McGraw–Hill, New York, 1994, pp. 146–156.

[10]Archer, R. D., and Saarlas, M., *An Introduction to Aerospace Propulsion*, 1st ed., Prentice–Hall, Upper Saddle River, NJ, 1996, pp. 68, 69.

[11]Yee, H. C., "A Class of High-Resolution Explicit and Implicit Shock-Capturing Methods," NASA TM 101088, NASA Ames Research Center, Feb. 1989.

[12]Buning, P. G., and Steger, J. L., "Solution of the Two-Dimensional Euler Equations with Generalized Coordinate Transformations Using Flux Splitting," AIAA Paper 82-0971, 1982.